

Design and Performance of Ion-Engine Systems for Control of Earth Satellites

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The application and optimization of ion-propulsion systems for orbit control of earth satellites was studied. These studies consisted of an evaluation of control requirements of various satellite systems and a design analysis of optimized ion engine systems that satisfy these requirements. A computer program was developed to determine the motion of earth satellites under the influence of perturbing forces such as the asphericity of the earth, the gravitational attraction of the sun and moon, atmospheric drag, solar pressure, and corrective thrust. From these data conceptual designs of ion engine control systems were performed. The applications considered were synchronous satellites, nonsynchronous medium altitude satellites, and manned space stations.

Nomenclature

A	= earth's oblateness coefficient
A_d	= satellite drag area
A_s	= satellite area exposed to solar radiation
c	= speed of light in a vacuum
C_d	= satellite coefficient of atmospheric drag
i_0	= initial inclination of the satellite orbit
K^2	= earth's gravitational constant
K_1^2	= moon's gravitational constant
K_s^2	= sun's gravitational constant
m_s	= satellite mass
r	= initial orbital radius
R	= moon's distance from the center of the earth
R_s	= sun's distance from the center of the earth
T	= earth triaxiality coefficient
V	= satellite orbital velocity
β	= angular position of the moon as measured in the satellite plane from the line of nodes
γ	= angular position of the sun as measured in the satellite plane from the line of nodes
ϵ	= coordinate axis normal to the initial satellite plane
ν	= angular position of the satellite as measured in the satellite plane from the line of nodes
ξ	= coordinate axis through the satellite line of nodes and assumed to coincide with the vernal equinox direction
ρ	= density of atmospheric particles
τ	= moon's latitude with respect to the satellite plane
u	= coordinate axis in the initial satellite plane which is normal at the ξ axis
ϕ	= radiant flux from the sun at the earth's distance
ψ	= sun's latitude with respect to the satellite plane
ω_s	= nominal angular velocity of the satellite
Ω	= angular position of equator's minor axis measured from vernal equinox direction

Introduction

ION-propulsion attitude control and orbit maintenance systems offer significant advantages over competing systems when applied to the control of earth satellites. There are two primary reasons why ion engines are well

suited to satellite control missions. First, because of the high specific impulse capability of ion engines, the propellant weight for most applications becomes a small portion of the over-all control system weight (as compared to chemical rockets, for example). As the total impulse required by the mission increases, either by increasing the mass of the satellite or extending its desired lifetime, this propellant weight advantage becomes even more pronounced. Therefore, the ion-engine system will, in many satellite applications, emerge as the minimum weight control system. Second, an ion-engine system offers a degree of control in both the attitude control and orbit maintenance functions which is not attainable with the more conventional chemical rocket systems. A major advantage of an ion-engine attitude control system, for example, is the precise control of the minimum impulse bit afforded by the ion thrust device (e.g., thrust is initiated and terminated instantaneously by voltage shutdown and valve closure). Because of this capability, the angular deviation and the angular drift rate of a satellite as well as the propellant utilization can be held to a minimum. In the case of orbit maintenance, the ion-engine system can be designed to continuously counteract the orbit perturbing forces, thereby preventing the satellite from deviating to any measurable degree at any time from its initial orbit.

There are also two primary reasons why satellite control is well suited to early ion-engine systems. First, for many uses the required thrust levels are on the order of millipounds or less. At these thrust levels, the primary power source is a relatively small percentage of the total system weight (as opposed, for example, to high-power prime propulsion systems), and a small increase in required power has only a slight weight effect. The ion engine can, therefore, be operated at conservative current densities and efficiencies, well within the state of the art, without seriously degrading the competitive position of the control system. In addition, operation at these moderate current densities gives even present-day engines predicted lifetimes in excess of most mission requirements. In fact, ion engines with thrust levels quite suitable for such satellite control applications have already been flight tested. Second, since the required power levels are, in general, relatively low, the primary power can be derived from solar cells. The obvious advantage here is that these power sources are presently available.

In order to test the applicability of ion engines to the control of earth satellites, an ion-propulsion attitude control and station-keeping system for synchronous satellites has

Presented as Preprint 64-501 at the 1st AIAA Annual Meeting, Washington, D. C., June 29-July 2, 1964; revision received April 21, 1965.

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been developed.¹ This system, which represents the efforts of the first application-oriented program in the field of electric propulsion, has reached the laboratory prototype stage and is undergoing extensive laboratory tests at NASA's Lewis Research Center. This system can lead to flight-qualified hardware for actual space tests and finally for specific satellite applications.

Along with the hardware development effort, a study program was carried out to optimize the design of the ion-propulsion synchronous satellite control system. As a result the station-keeping system concept was modified to include a continuous² rather than pulsed mode of operation. These studies were extended to determine the applicability of the technology developed during the hardware program to satellite control missions other than those involving synchronous satellites. This paper presents the results of these studies including an evaluation of the control requirements of various satellite systems and a design of optimized ion-engine systems that satisfy these requirements.

Perturbing Accelerations

The motion of a satellite revolving about the earth does not follow simply from a solution of the classical two-body problem. There are forces which, although small compared to the primary central force field, perturb the motion of an earth satellite and modify its initial orbit. The major perturbing accelerations can be separated and classified in terms of their sources as follows: 1) asphericity of the earth, a) oblateness and b) triaxiality; 2) gravitational attraction of celestial bodies, a) sun and b) moon; 3) solar radiation pressure; 4) atmospheric drag. Other causes of orbit perturbation such as the pear-shapeness of the earth, electric and magnetic fields, meteorite collisions, and relativistic effects are normally negligible compared to those listed previously and will not seriously affect the control-system design. The principal causes of satellite perturbation at relatively low orbital altitudes are atmospheric drag and the asphericity of the earth. At the greater orbital altitudes, the perturbing effects that dominate are those due to lunar and solar attractive forces and solar radiation pressure. Figure 1 illustrates the relative acceleration levels due to the major sources of orbit perturbation on an earth satellite as a function of orbit radius. The modifications in satellite orbit that can result³ from these perturbing forces include variations in orbital elements such as orbital period, eccentricity, inclination, semimajor axis as well as effects such as rotation

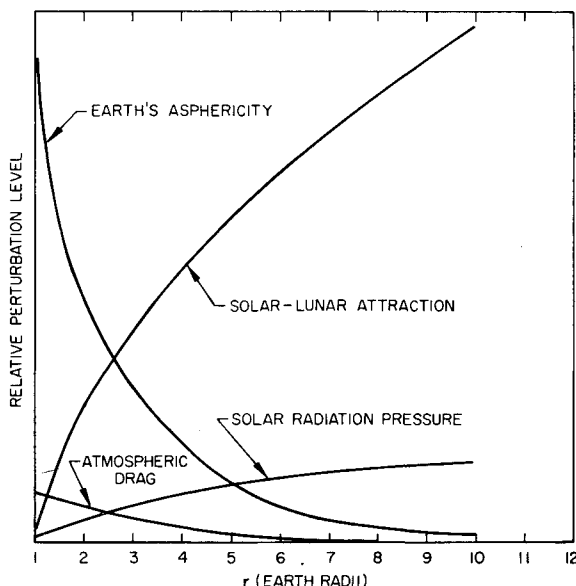


Fig. 1 Relative perturbing accelerations vs orbit radius.

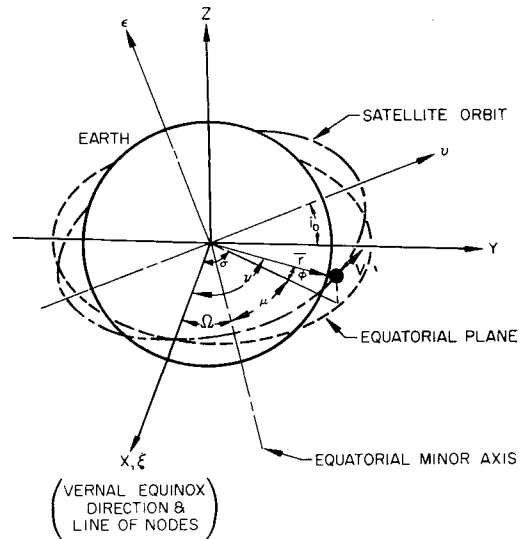


Fig. 2 Geocentric coordinate system.

of the nodal line and rotation of the major axis. These modifications of the Keplerian orbit can be periodic, continuous, or constant. For this reason, the curves in Fig. 1 do not necessarily represent the relative requirements placed on a control system, since, for example, a periodic effect may not have to be corrected, if the maximum excursion of the satellite is within the limits acceptable. In addition, the effects classified as constant can be corrected by initial adjustment of the desired orbit.

Assuming that the initial satellite orbit is circular, the equations of motion (Fig. 2) of a satellite, including the effects of perturbations, are then symbolically

$$\ddot{\mathbf{r}} = (\ddot{\mathbf{r}})_p = (\ddot{\mathbf{r}})_{a1} + (\ddot{\mathbf{r}})_{a2} + (\ddot{\mathbf{r}})_{sl} + (\ddot{\mathbf{r}})_{sp} + (\ddot{\mathbf{r}})_d \quad (1)$$

$$\ddot{\epsilon} + \omega_s^2 \epsilon = (\ddot{\epsilon})_p = (\ddot{\epsilon})_{a1} + (\ddot{\epsilon})_{a2} + (\ddot{\epsilon})_{sl} + (\ddot{\epsilon})_{sp} + (\ddot{\epsilon})_d \quad (2)$$

$$\ddot{r} + \omega_s^2 r = (\ddot{r})_p = (\ddot{r})_{a1} + (\ddot{r})_{a2} + (\ddot{r})_{sl} + (\ddot{r})_{sp} + (\ddot{r})_d \quad (3)$$

where the subscripts are p , total perturbation; $a1$, oblateness of earth; $a2$, triaxiality of earth; sl , solar-lunar attraction; sp , solar pressure; d , atmospheric drag; and ω_s , angular velocity of the satellite. It will also be assumed that for a given satellite application only minor variations of the orbital elements are allowed to occur before a correction is made. With this assumption, the perturbing accelerations can be approximated by a function involving position angles only which results in the linearization of the equations of motion. Hence, ω_s , r , and i are taken as constants for each satellite orbit considered. One result of the linearization of the equations of motion is that the radial motion of a satellite due to a tangential component of force is directly obtainable by solving Eq. (1) and applying Newton's gravitational law to the results. Also, since the radial components of the perturbing accelerations are normally negligible compared to the principal radial acceleration (i.e., the gravitational attraction of the earth), Eq. (3) will not be considered here. In addition, the study is limited to a consideration of first-order perturbation terms, since these are the only ones of prime importance in control-system design. The problem then becomes one of developing analytical expressions for $(\ddot{\mathbf{r}})_p$ and $(\ddot{\epsilon})_p$ in terms of the satellite, moon, and sun position angles and the position of the equatorial minor axis and of solving these equations while incorporating corrective thrust modes as desired. Although the perturbed motion could be determined in closed form, the application of thrust which is performed in a nonlinear fashion necessitates the use of a computer. The computer program,⁴ therefore, provides 1) a description of the perturbed motion of earth satellites, 2) an evaluation of satellite-

pellant requirements. These weights go up rapidly as the total impulse (or mission time) increases. The weight of the ion-engine system, on the other hand, is determined almost entirely by the initial hardware, and it increases very slowly (approximately 2 lbs/yr for a 1000-lb vehicle) with required operation time. The curves in Fig. 5 show that, as the desired lifetime of satellites extends to periods of years, the ion-propulsion system emerges as the lightest attitude control and station-keeping system available. When operating in the semicontinuous correction mode, the break-even point on weight is six months to one year, probably the minimum desirable lifetime for communication or meteorological satellites. The weight of the ion-engine system designed for station keeping only is also shown.

Nonsynchronous Communication Satellite System

The communication between ground stations via satellite relays has been studied in great detail. Three possible satellite systems, which could provide world-wide communication coverage and which have been given the most attention, are synchronous satellites, nonsynchronous randomly distributed satellites, and nonsynchronous evenly spaced satellites. World-wide coverage could be attained essentially with as few as three synchronous satellites. In the case of nonsynchronous satellites, a larger number of satellites are required to provide an equivalent coverage. The exact number is given in a study⁷ performed at the Aerospace Corporation. Various combinations of nonsynchronous satellites (all of which were assumed to be in 5000-naut-mile circular orbits) were studied. Assuming a criterion that the probability of being able to communicate between any two terminals in a network of 25 ground stations must be approximately unity, two orbital systems proved attractive. The first system consisted of 30 satellites with 12 randomly distributed in the equatorial plane and 6 distributed in each of 3 equally separated polar planes. The second system consisted of only 16 satellites with 8 evenly spaced in an equatorial orbit and 8 evenly spaced in a polar orbit. Obviously from these results the evenly spaced satellite system offers the distinct advantage of reducing the number of satellites required for world-wide coverage to about one-half of those needed in the randomly distributed system. In order to maintain the proper spacing, the evenly spaced system would require a station-keeping capability.

The principal perturbing forces on a 5000-naut-mile earth satellite in either an equatorial or polar orbit are those due to the triaxiality of the earth and the solar-lunar attraction. Compared to these, effects of atmospheric drag and solar radiation pressure are negligible. (It is assumed that the surface area to mass ratio for these satellites is typical of high-density electronic systems.) In addition, the perturbation due to the oblateness of the earth is a function of orbital inclination and is zero for both equatorial and polar orbits.

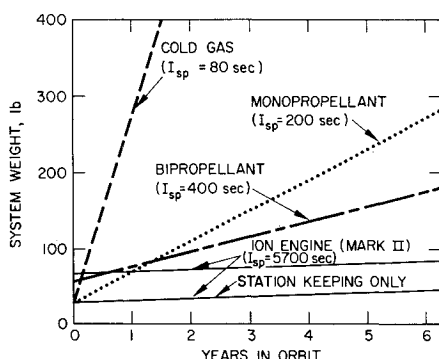


Fig. 5 Comparison of the weights of several types of attitude and station-keeping systems for use with a 1000-lb vehicle.

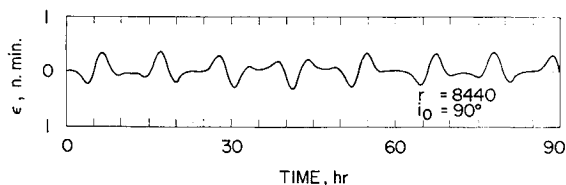


Fig. 6 Motion of 5000-naut-mile polar satellite normal to initial orbit due to triaxiality effect.

The perturbing acceleration due to the triaxiality of the earth has no component normal to an equatorial orbit. Components do exist, however, in the tangential direction for a satellite in an equatorial orbit and in both the normal and tangential directions for the polar-orbit case. The tangential component in the equatorial case results in a periodic oscillation around the nominal value of the orbital velocity, so that there is a maximum variation from the desired arc position of about 50 ft. The period of this cycle is 3.5 hr when the satellite revolves in the same direction as the earth rotates and 2.2 hr when revolving in the opposite direction. The effect of the normal component on the motion of a polar satellite is illustrated in Fig. 6. The variation in ϵ is small and periodic. The period of each cycle is about 60 hr, but the motion during the cycle is rather unique. This results from the highly involved form of the driving function [Eq. (6)]. The tangential effect of triaxiality on a polar satellite presents a very different situation. Figure 7 illustrates this motion for the medium-attitude satellite. The net result is a change in nominal orbit position which is directly proportional to time. Hence, $(r\Delta\nu)_{a2} = \text{const} + \text{cosine wave}$. The constant term can be eliminated by an appropriate correction in the initial orbit radius. The remaining term is, of course, periodic and has a relatively small amplitude, so that no correction would be required.

Solar-lunar attraction will produce normal and tangential forces on both the equatorial and polar satellite systems. The tangential effects on these satellites are of the same form as appeared in Fig. 7. Hence a similar type of correction is implied, i.e., a Δr change, to eliminate this motion. Figures 8a and 8b illustrate the normal effect of solar-lunar attraction on equatorial and polar orbit satellites, respectively. In the cases shown, the perturbing forces are at their annual maximum. The perturbed motion shown represents a gradual increase in inclination for the equatorial satellite and a rotation about a line in the orbit plane for the polar satellite. Figures 8a and 8b also show the corrected motion of the satellite system. Continuous corrective accelerations of $6.5 \times 10^{-6} g$ and $5.9 \times 10^{-6} g$ are required for the 0° and 90° inclined orbits, respectively. Since, however, the perturbing forces vary as sine waves with a major period of one year, the average rate of increase will be less by a factor of 0.636 than shown in Figs. 8a and 8b. The values for $|a_c|$ are also reduced by the same factor (viz: $4.1 \times 10^{-6} g$ for the equatorial satellites and $3.8 \times 10^{-6} g$ for the polar satellites).

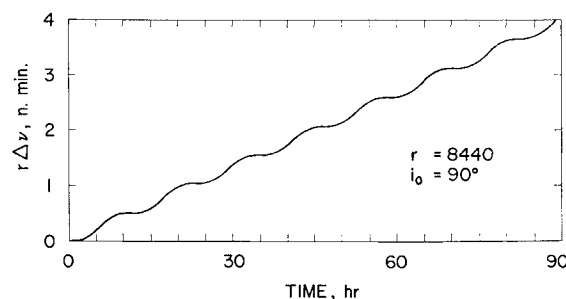


Fig. 7 Motion of 5000-naut-mile polar satellite tangent to initial orbit due to triaxiality effects.

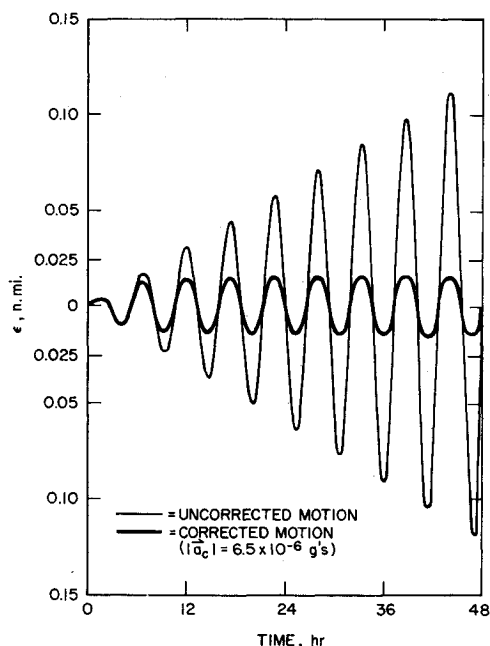


Fig. 8a Motion of 5000-naut-mile equatorial satellite normal to initial orbit due to solar-lunar effect.

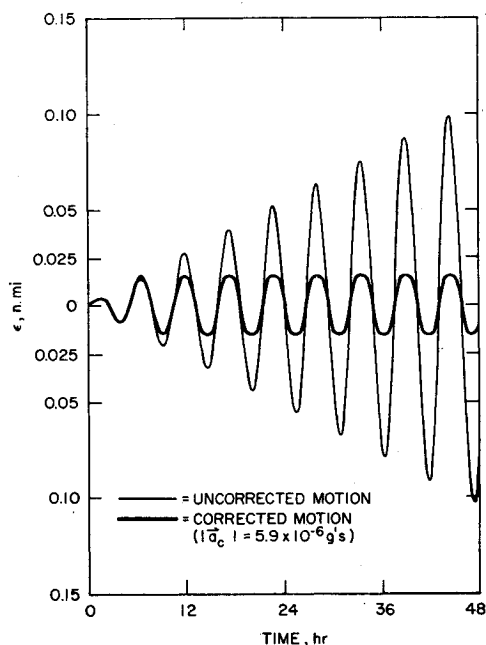


Fig. 8b Motion of 5000-naut-mile polar satellite normal to initial orbit due to solar-lunar effect.

It can be concluded that the orbit maintenance system for medium-altitude satellites in equatorial or polar orbits should be designed primarily to counteract the solar-lunar perturbations. Since these forces act in a direction perpendicular to the orbital plane, the control system must provide thrust vectors normal to the satellite motion. Corrective thrust is most efficient when applied at the satellite crossings of the initial orbit. However, in the case of ion propulsion the lowest power requirement occurs when thrust is applied continuously. The application of continuous thrust would require a control system consisting of two ion engines whose thrust vectors were normal to the orbit plane and in opposite directions. These engines would fire alternately (changing at the peaks of the oscillations shown in Figs. 8a and 8b) always opposing the normal component of the satellite velocity. For satellites in the 500-lb class, a

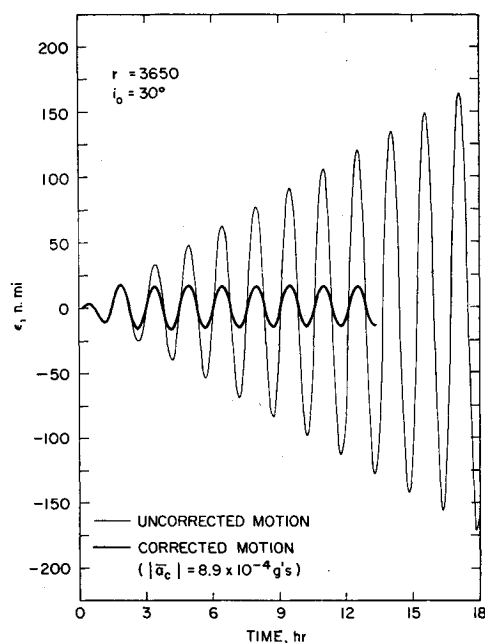


Fig. 9 Motion of 210-naut-mile, 30° inclined satellite normal to initial orbit due to oblateness effect.

thrust level of about 2 mlb is required for orbit maintenance.

An ion engine, which is presently available and which is nominally rated for thrust levels of 1.5–5 mlb, is shown in Fig. 4 as the station-keeping engine for the synchronous satellite control system. Two of these thrusters along with their feed systems, an engine control system, power conditioning equipment, and a primary power supply could make up the complete control system. The system specifications (nominal) are as follows: 1) thrust level, 2 mlb; 2) specific impulse, 5000 sec; 3) power requirement, 400 W; 4) engine-system weight (including thrusters, feed systems, controls, and power conditioning), 35 lb; 5) propellant weight per year, 15 lb; 6) size (including propellant): diameter, 4 in.; and length, 8 in.

The weight of solar cells to provide the required power for the ion-engine control system is on the order of 60 lb, resulting in an over-all system weight of under 100 lb. The propellant alone for a chemical-rocket system, performing the same mission, would weigh more than 150 lb. For a two-year satellite lifetime, the ion-engine system would increase in weight to 110 lb, whereas the chemical-rocket system would weigh over 300 lb.

Manned Space Station

The establishment of large manned orbiting space stations is presently under consideration. Several configurations

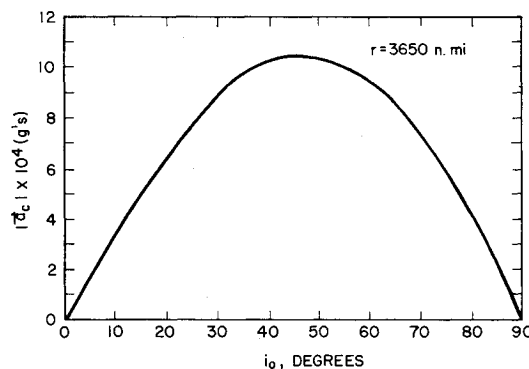


Fig. 10 Effect of inclination on required continuous corrective thrust (oblateness effect).

have already been put forth with proposed lifetimes ranging from 1 to 5 years. These space stations will be able to serve as laboratories to evaluate the capabilities of man in a space environment, to perform basic and applied research, and to gather astronomical data, or as bases for orbital launch, logistics, and maintenance support missions. For most of these uses, it will be desirable to maintain the initial orbit of the manned satellite. An onboard propulsion system will be required, therefore, to counteract the various forces that tend to perturb the orbit. Typical of the advanced manned laboratories is a 200,000-lb space station located in a 210-naut-mile circular orbit at an inclination of 30° .

As in the case of the nonsynchronous satellites, certain of the principal perturbations will have negligible effects on the manned space station. At an altitude of 210-naut-miles, the perturbing acceleration due to the earth's oblateness is many times greater than solar pressure, solar-lunar attraction, triaxiality, and atmospheric drag. Atmospheric drag, although relatively small in acceleration level, does, however, produce a major effect in the tangential direction.

The effect of oblateness in the tangential direction is similar to that shown in Fig. 7 and can be corrected by adjustment of the initial orbit radius. The motion of the space station normal to the initial 30° inclined orbit is presented in Fig. 9. Although these results are of a form similar to those in Figs. 8a and 8b, the motion is somewhat different. Figures 8a and 8b represent a deviation from the initial orbit due to a rotation about a line in the orbit plane. Figure 9, on the other hand, results from a rotation of the orbit about the polar axis, that is a precession of the satellite orbit with respect to the equatorial plane. (There is an inclination change included in this motion but the resulting deviation in ϵ is small and periodic.) The rotation of the line of nodes, which is an outward manifestation of this orbital precession, is found to be $\sim \frac{1}{2}^\circ$ per satellite revolution. This motion can be arrested (or modified) as shown in Fig. 9 by application of a continuous corrective acceleration on the order of 10^{-3} g. These acceleration levels are, of course, not within the capabilities of an ion-propulsion system. It should be noted that, since the earth rotates through about 22° per satellite revolution, the correction or enhancement of this perturbation would not appreciably alter the path of a space station over a given land mass and, therefore, would be of questionable value. The results in Fig. 9 are typical for a wide range of satellite orbit radii and inclinations.

The magnitude of the required continuous corrective acceleration, $|a_c|$ is shown in Fig. 10 as a function of orbit inclination. The oblateness effect is seen to vary from zero in the equatorial plane to a maximum at an inclination of 45° to zero in the polar plane. The effect of orbital radius on $|a_c|$ is illustrated in Fig. 11. This curve shows that the acceleration level decreases from 10^{-3} to 10^{-4} g over an altitude range of 100-naut-miles to 1800-naut-miles.

Atmospheric drag, which is assumed to have no normal component (i.e., orbital precession is neglected), produces

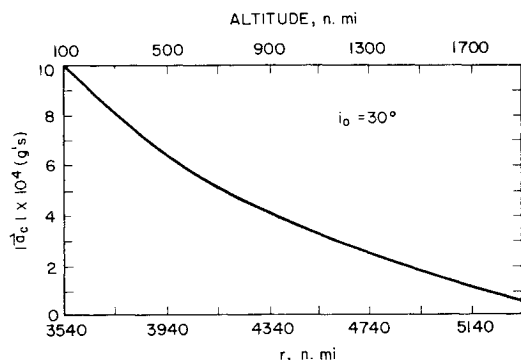


Fig. 11 Effect of orbital radius on required continuous corrective thrust (oblateness effect).

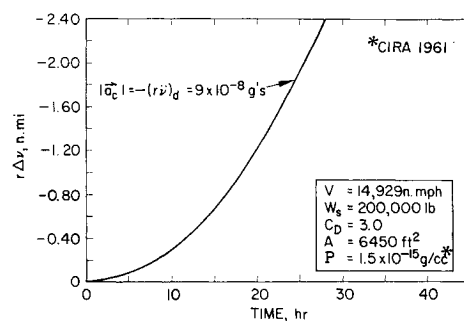


Fig. 12 Motion of 210-naut-mile satellite tangent to initial orbit due to atmospheric drag.

the only significant tangential perturbation on a low-altitude satellite. The effect of drag on the tangential motion of a space station with a ballistic coefficient W_s/AC_d of 10 lb/ft² and at an altitude of 210-naut-miles is given in Fig. 12. The continuous corrective acceleration required to maintain the initial orbit is found to be 9×10^{-8} g. The data, presented in Fig. 12, assume that the atmosphere is stationary with respect to an inertial reference frame and that its density is a function of altitude only.

A control system designed to maintain the orbit of an earth satellite in a 200- to 300-naut-mile orbit can be limited to counteracting the perturbing force due to atmospheric drag. The magnitude of this perturbing force in the case of a typical manned space station is on the order of 20 mlb. Since the drag forces act in a direction opposite to the direction of motion, corrective thrust is most efficient when applied tangent to the satellite orbit in the direction of the satellite velocity vector. A 20-mlb thruster vectored through the center of gravity in the direction of vehicle motion will, therefore, by continuous operation maintain the orbital altitude of a space station.

Because of the inevitable limitations in available power, the low-thrust ion-engine system must operate continuously to provide the total corrective impulse required. The power and propellant requirements for an ion-engine orbit maintenance system are shown as a function of drag force and total impulse in Fig. 13. For a drag force of 20 mlb, about 4 kw of power is needed to operate the propulsion system. The corresponding propellant weight for a one-year operation is 125 lb. For a similar period, the chemical rocket would require 1600 lb, or more, of propellant.

Since the ion engine must thrust continuously through the center of gravity of the vehicle and tangent to the orbit, an additional requirement of position mechanization is imposed on the system, if the vehicle is sun-oriented. Obviously, if the vehicle is orbit-oriented no such directional control is required, and the engine station can be rigidly attached to the vehicle. The system specifications, based on thrusters presently under development, are as follows: 1) thrust level,

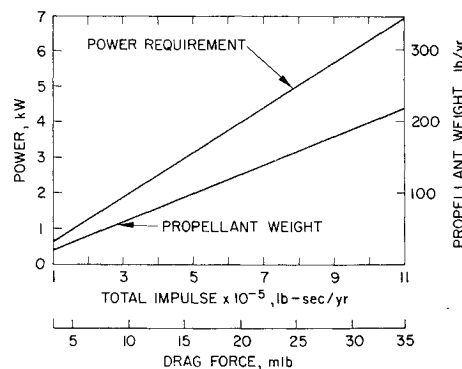


Fig. 13 Ion-propulsion orbit maintenance system.

20 mlb (nominal); 2) specific impulse, 5000 sec; 3) power requirement, 4 kw; 4) system weight, 98 lb; 5) propellant weight per 180 days (i.e., possible resupply period) 60 lb; 6) size (including propellant): diameter, 12 in.; and length, 24 in.

Conclusions

The high specific impulse ion engine has been shown to be strongly competitive with chemical rockets for applications such as the attitude control and station keeping of synchronous satellites. The ion-propulsion control system offers advantages over other contemporary control systems both in total system weight and precision control. These advantages extend to other satellite control applications such as orbit maintenance for nonsynchronous medium-altitude satellites and manned space stations. For most satellite control applications, the power required to operate the ion engine is less than a few kilowatts (usually on the order of 100-1000 w) and can be supplied, therefore, by solar cells. Ion-engine efficiencies and lifetimes have already reached levels that satisfy most satellite control requirements. Furthermore, demonstration of an operating prototype of a satellite control system has shown that such systems can be reduced to engineering practice. For these reasons, ion pro-

pulsion can be given serious consideration for near-future satellite control applications.

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SEPT.-OCT. 1965

J. SPACECRAFT

VOL. 2, NO. 5

Development Status of Low-Power Arc-Jet Engines

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The results of a continuous performance test of a 2-kw arc-jet thruster are presented. The design concepts that were employed to obtain the thruster design are covered in some detail. This thruster is a revised version of an earlier 1-kw thruster design. In addition, the results of a pulsed-mode test on a 1-kw thruster are presented. A continuous lifetime of 150 hr without failure of the thruster has been demonstrated. This same thruster has been operated at specific impulse levels as high as 1270 sec and at input power levels up to 3.5 kw. In the pulsed-mode operation, 17,580 hot thrust pulses were achieved in the test.

Nomenclature

a, A	= area, m ² , ft ²
F	= thrust, lb, newton
f	= friction force, newton
i	= specific enthalpy, joule/kg°C
I_{sp}	= specific impulse, sec
m	= $\tan \gamma$
\dot{m}	= mass flow, lb/sec, kg/sec
M	= momentum, newton
p	= pressure, newton/m ² , in Hg
R, U, V, W	= dimensionless parameters, Eq. (3)
Re	= Reynolds number
r	= radius, m
t	= time, sec
T	= temperature, °K
u	= velocity, m/sec, fps

v	= specific volume, m ³ /kg
x, y, ξ, η	= Cartesian coordinates
γ	= angle, isentropic exponent
ϵ	= momentum ratio [see Eq. (19)]
λ_f	= friction coefficient
ρ	= density, kg/m ³
μ	= absolute viscosity, kg/sec-m
θ	= nozzle divergence half angle
ψ	= density ratio, ρ/ρ_e
A^*	= dimensionless parameter, Eq. (5)
ϕ, Ω	= dimensionless parameters, Eq. (13)

Subscripts

c	= nozzle chamber
e	= nozzle exit
is	= isentropic
o	= origin
st	= stagnation
th	= nozzle throat

Introduction

STUDIES have shown that electrothermal thrusters, typified by the arc-jet and resistojet, are suitable for a variety of early space applications.^{1,2} This paper considers

Presented as Preprint 64-502 at the 1st AIAA Annual Meeting, Washington, D. C., June 29-July 2, 1964; revision received March 4, 1965. Most of the analytical and experimental results presented herein were obtained under the sponsorship of NASA Lewis Research Center, Contract No. NAS 3-2521.

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